CHAPTER 3

Simulation of Business Processes

The stock and flow diagram which have been reviewed in the preceding two chapters show more about the process structure than the causal loop diagrams studied in Chapter 1. However, stock and flow diagrams still don t answer some important questions for the performance of the processes. For example, the stock and flow diagram in Figure 2.1b shows more about the process structure than the causal loop diagram in Figure 2.1a, but it still doesn t answer some important questions. For example, how will the number of Potential Customers vary with time? To answer questions of this type, we must move beyond a graphical representation to consider the *quantitative* features of the process. In this example, these features include such things as the initial number of Potential and Actual Customers, and the specific way in which the sales flow depends on Potential Customers.

When deciding how to quantitatively model a business process, it is necessary to consider a variety of issues. Two key issues are how much detail to include, and how to handle uncertainties. Our orientation in these notes is to provide tools that you can use to develop better insight about key business processes. We are particularly focusing on the intermediate level of management decision making in an organization: Not so low that we must worry about things like specific placement of equipment in an manufacturing facility, and not so high that we need to consider decisions that individually put the company at risk.

This intermediate level of decision is where much of management s efforts are focused, and improvements at this level can significantly impact a company s relative competitive position. Increasingly, these decisions require a cross-functional perspective. Examples include such things as the impact on sales for a new product of capacity expansion decisions, relationships between financing and production capacity decisions, and the relationship between personnel policies and quality of service. Quantitatively considering this type of management decision may not require an extremely detailed model for business processes. For example, if you are considering the relationship between personnel policies and quality of service, it is probably not necessary to consider individual workers with their pay rates and vacation schedules. A more aggregated approach will usually be sufficient.

Furthermore, our primary interest is improving existing processes which are typically being managed intuitively, and to make these improvements in a reasonable amount of time with realistic data requirements. There is a long history of efforts to build highly detailed models, only to find that either the data required are not available, or the problem addressed has long since been solved by other means before the model was completed. Thus, we seek a relatively simple, straightforward quantitative modeling approach which can yield useful results in a timely manner.

The approach we take, which is generally associated with the field of system dynamics (Morecroft and Sterman 1994), makes two simplifying assumptions: 1) flows within processes are continuous, and 2) flows do not have a random component. By continuous flows, we mean that the quantity which is flowing can be infinitely finely divided, both with respect to the quantity of material flowing and the time period over which it flows. By not having a random component, we mean that a flow will be exactly specified if the values of the variables at the other end of information arrows into the flow are known. (A variable that does not have a random component is referred to as a *deterministic* variable.)

Clearly, the continuous flow assumption is not exactly correct for many business processes: You can t divide workers into parts, and you also can t divide new machines into parts. However, if we are dealing with a process involving a significant number of either workers or machines, this assumption will yield fairly accurate results and it substantially simplifies the model development and solution. Furthermore, experience shows that even when quantities being considered are small, treating them as continuous is often adequate for practical analysis.

The assumptions of no random component for flows is perhaps even less true in many realistic business settings. But, paradoxically, this is the reason that it can often be made in an analysis of business processes. Because uncertainty is so widely present in business processes, many realistic processes have evolved to be relatively insensitive to the uncertainties. Because of this, the uncertainty can have a relatively limited impact on the process. Furthermore, we will want any modifications we make to a process to leave us with something that continues to be relatively immune to randomness. Hence, it makes sense in many analyses to assume there is no uncertainty, and then test the consequences of possible uncertainties.

Practical experience indicates that with these two assumptions, we can substantially increase the speed with which models of business processes can be built, while still constructing models which are useful for business decision making.

3.1 Equations for Stocks

With the continuous and deterministic flow assumptions, a business process is basically modeled as a plumbing system. You can think of the stocks as tanks full of a liquid, and the flows as valves, or, perhaps more accurately, as pumps that control the rate of flow between the tanks. Then, to completely specify the equations for a process model you need to give 1) the initial values of each stock, and 2) the equations for each flow.

We will now apply this approach to the advertising stock and flow diagram in Figure 2.1b. To do this, we will use some elementary calculus notation. However, fear not! You do not have to be able to carry out calculus operations to use this approach. Computer methods are available to do the required operations, as is discussed below, and the calculus discussion is presented for those who wish to gain a better understanding of the theory behind the computer methods.

The number of Potential Customers at any time t is equal to the number of Potential Customers at the starting time minus the number that have flowed out due to sales. If sales is measured in customers per unit time, and there were initially 1,000,000 Potential Customers, then

$$\text{Potential Customers}(t) = 1\ 000\ 000 - \int_0^t \text{sales}(\tau)\, d\tau \qquad (3\ 1)$$

where we assume that the initial time is $t = 0$, and τ is the dummy variable of integration. Similarly, if we assume that there were initially zero Actual Customers, then

$$\text{Actual Customers} = \int_0^t \text{sales}(\tau)\, d\tau \qquad (3\ 2)$$

The process illustrated by these two equations generalizes to any stock: The stock at time t is equal to the initial value of the stock at time $t = 0$ plus the integral of the flows into the stock minus the flows out of the stock. Notice that once we have drawn a stock and flow diagram like the one shown in Figure 2.1b, then a clever computer program could enter the equation for the value of any stock at any time without you having to give any additional information except the initial value for the stock. In fact, system dynamics simulation packages automatically enter these equations.

3.2 Equations for Flows

However, you must enter the equation for the flows yourself. There are many possible flow equations which are consistent with the stock and flow diagram in Figure 2.1b. For example, the sales might be equal to 25,000 customers per month until the number of Potential Customers drops to zero. In symbols,

$$\text{sales}(t) = \begin{cases} 25\ 000 & \text{Potential Customers}(t) > 0 \\ 0 & \text{otherwise} \end{cases} \qquad (3\ 3)$$

A more realistic model might say that if we sell a product by advertising to Potential Customers, then it seems likely that some specified percentage of Potential Customers will buy the product during each time unit. If 2.5 percent of the Potential Customers make a purchase during each month, then the equation for sales is

$$\text{sales}(t) = 0\ 025 \times \text{Potential Customers}(t) \qquad (3\ 4)$$

(Notice that with this equation the initial value for sales will be equal to 25,000 customers per month.)

3.3 Solving the Equations

If you are familiar with solving differential equations, then you can solve equations 3.1 and 3.2 in combination with either equation 3.3 or equation 3.4 to obtain a graph of Potential Customers over time. However, it quickly becomes infeasible to solve such equations by hand as the number of stocks and flows increases, or if the equations for the stocks are more complex than those shown in equation 3.3 and 3.4. Thus, computer solution methods are almost always used.

We will illustrate how this is done using the Vensim simulation package. With this package, as with most PC-based system dynamics simulation systems, you typically start by entering a stock and flow diagram for the model. In fact, the stock and flow diagram shown in Figure 2.1b was created using Vensim. You then enter the initial values for the various stocks into the model, and also the equations for the flows. Once this is done, you then tell the system to solve the set of equations. This solution process is referred to as *simulation*, and the result is a time-history for each of the variables in the model. The time history for any particular variable can be displayed in either graphical or tabular form.

Figure 3.1 shows the Vensim equations for the model using equations 3.1 and 3.2 for the two stocks, and either equation 3.3 (in Figure 3.1a) or equation 3.4 (in Figure 3.1b) for the flow sales. These equations are numbered and listed in alphabetical order.

Note that equation 1 in either Figure 3.1a or b corresponds to equation 3.2 above, and equation 4 is either Figure 3.1a or b corresponds to equation 3.1 above. These are the equations for the two stock variables in the model. The notation for these is straightforward. The function name `INTEG` stands for integration, and it has two arguments. The first argument includes the flows into the stock, where flows out are entered with a minus sign. The second argument gives the initial value of the stock.

Equation 5 in Figure 3.1a corresponds to equation 3.3 above, and equation 5 in Figure 3.1b corresponds to equation 3.4 above. These equations are for the flow variable in the model, and each is a straightforward translation of the corresponding mathematical equation.

Equation 3 in either Figure 3.1a or b sets the lower limit for the integrals. Thus, the equation `INITIAL TIME = 0` corresponds to the lower limits of $t = 0$ in equations 3.1 and 3.2 above. Equation 2 in either Figure 3.1a or b sets the last time for which the simulation is to be run. Thus, with `FINAL TIME = 100`, the values of the various variables will be calculated from the `INITIAL TIME` (which is zero) until a time of 100 (that is, $t = 100$).

Equations 6 and 7 in either Figure 3.1a or b set characteristics of the simulation process.

```
(1)  Actual Customers = INTEG( sales , 0)
(2)  FINAL TIME = 100
(3)  INITIAL TIME = 0
(4)  Potential Customers = INTEG( - sales , 1e+006)
(5)  sales = IF THEN ELSE ( Potential Customers > 0, 25000, 0)
(6)  SAVEPER = TIME STEP
(7)  TIME STEP = 1
```

a. Equations with constant sales

```
(1)  Actual Customers = INTEG( sales , 0)
(2)  FINAL TIME = 100
(3)  INITIAL TIME = 0
(4)  Potential Customers = INTEG( - sales , 1e+006)
(5)  sales = 0.025 * Potential Customers
(6)  SAVEPER = TIME STEP
(7)  TIME STEP = 1
```

b. Equations with proportional sales

Figure 3.1 *Vensim equations for advertising model*

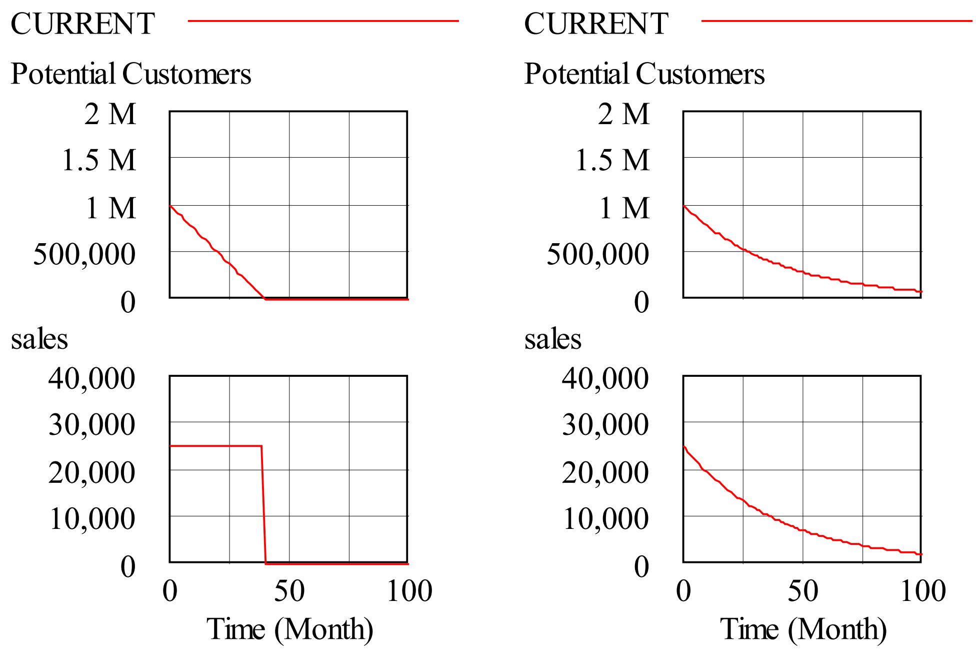
3.4 Solving the Model

The time histories for the sales and Potential Customers variables are shown in Figure 3.2. The graphs in Figure 3.2a were produced using the equations in Figure 3.1a, and the graphs in Figure 3.2b were produced using the equations in Figure 3.1b. We see from Figure 3.2a that sales stay at 25,000 customers per month until all the Potential Customers run out at time $t = 40$. Then sales drop to zero. Potential Customers decreases linearly from the initial one million level to zero at time $t = 40$. Although not shown in this figure, it is easy to see that Actual Customers must increase linearly from zero initially to one million at time $t = 40$.

In Figure 3.2b, sales decreases in what appears to be an exponential manner from an initial value of 25,000, and similarly Potential Customers also decreases in an exponential manner. (In fact, it can be shown that these two curves are exactly exponentials.)

3.5 Some Additional Comments on Notation

In the Figure 2.1b stock and flow diagram, the two stock variables Potential Customers and Actual Customers are written with initial capital letters on each word. This is recommended practice, and it will be followed below. Similarly, the flow sales is written in all lower case, and this is recommended practice.



a. Constant sales   b. Proportional sales

Figure 3.2 *Time histories of sales and Potential Customers*

While stock and flow variables are all that are needed in concept to create any stock and flow diagram, it is often useful to introduce additional variables to clarify the process model. For example, in a stock and flow diagram for the Figure 3.1b model, it might make sense to introduce a separate variable name for the sales fraction (which was given as 0.025 in equation 3.4. This could clarify the structure of the model, and also expedites sensitivity analysis in most of the system dynamics simulation packages.

Such additional variables are called *auxiliary variables*, and examples will be shown below. It is recommended that an auxiliary variable be entered in ALL CAPITAL LETTERS if it is a constant. Otherwise, it should be entered in all lower case letters just like a flow variable, except in one special case. This is the case where the variable is not a constant but is a prespecified function of time (for example, a sine function). In this case, the variable name should be entered with the FIRst three letters capitalized, and the remaining letters in lower case.

This notation allows you to quickly determine important characteristics of variable in a stock and flow diagram from the diagram without having to look at the equations that go with the diagram.

3.6 Reference

J. D. W. Morecroft and J. D. Sterman, editors, *Modeling for Learning Organizations*, Productivity Press, Portland, OR, 1994.