CHAPTER 4

Basic Feedback Structures

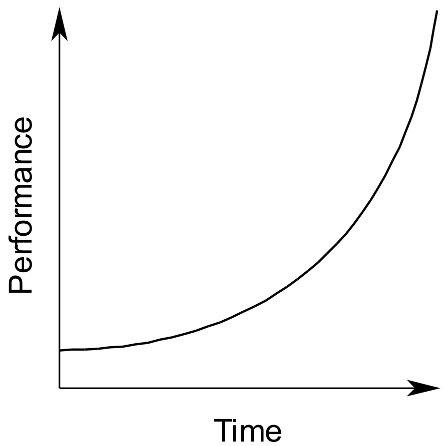
This chapter reviews some common patterns of behavior for business processes, and presents process structures which can generate these patterns of behavior. Many interesting patterns of behavior are caused, at least in part, by *feedback*, which is the phenomenon where changes in the value of a variable indirectly influence future values of that same variable. *Causal loop diagrams* (Richardson and Pugh 1981, Senge 1990) are a way of graphically representing feedback structures in a business process with which some readers may be familiar. However, causal loop diagrams only suggest the possible modes of behavior for a process. By developing a stock and flow diagram and corresponding model equations, it is possible to estimate the actual behavior for the process.

Figure 4.1 illustrates four patterns of behavior for process variables. These are often seen individually or in combination in a process, and therefore it is useful to understand the types of process structures that typically lead to each pattern.

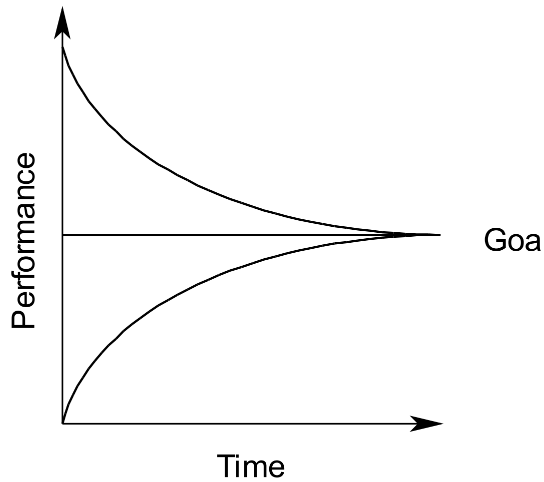
4.1 Exponential Growth

Exponential growth, as illustrated in Figure 4.1a, is a common pattern of behavior where some quantity feeds on itself to generate ever increasing growth. Figure 4.2 shows a typical example of this the growth of savings with compounding interest. In this case, increasing interest earnings lead to an increase in Savings, which in turn leads to greater interest because interest earnings are proportional to the level of Savings, as shown in equation 3 of Figure 4.2b. Figure 4.2c shows the characteristic upward-curving graph that is associated with this process structure. This is referred to as an exponential curve because it can be demonstrated that it follows the equation of the exponential function. (Remember that the cloud at the left side of Figure 4.2a means that we are not explicitly modeling the source of the interest.)

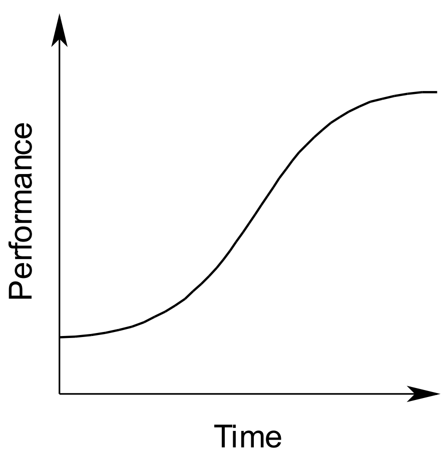
While it is possible to use standard calculus methods to solve for the variables in this model, we will not do this because this structure is typically only one



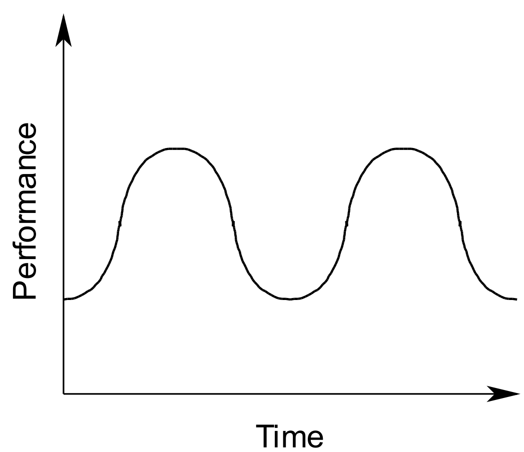
a. Exponential growth



b. Goal-seeking

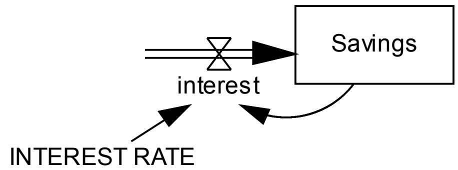


c. S-shaped



d. Oscillation

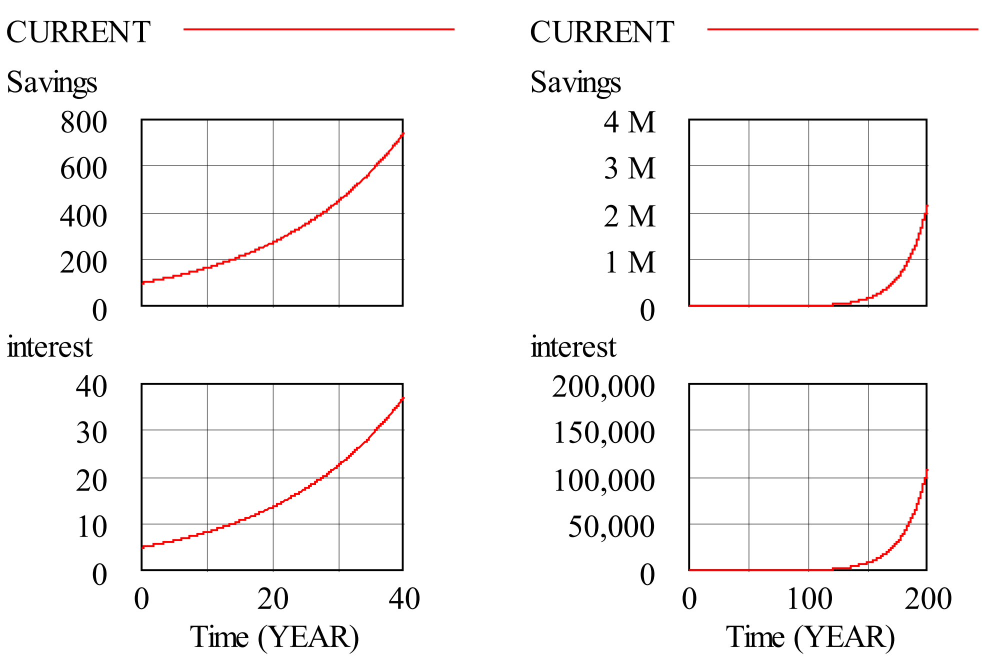
Figure 4.1 *Characteristic patterns of system behavior*



a. Stock and flow diagram

```
(1)  FINAL TIME  = 40
(2)  INITIAL TIME  = 0
(3)  interest = INTEREST RATE*Savings
(4)  INTEREST RATE = 0.05
(5)  SAVEPER  = TIME STEP
(6)  Savings = INTEG(interest,100)
(7)  TIME STEP  = 0.0625
```

b. Vensim equations



c. Forty year horizon

d. Two hundred year horizon

Figure 4.2 *Exponential growth feedback process* component of a more complex process in realistic settings. The models of those more complex processes usually cannot be solved in closed form, and therefore we have shown the Vensim simulation equations used to simulate this model in Figure 4.2b.

Figure 4.2d shows another characteristic of exponential growth processes. In this diagram, the time period considered has been extended to 200 years. When this is done, we see that exponential growth over an extended period of time displays a phenomenon where there appears to be almost no growth for a period, and then the growth explodes. This happens because with exponential growth the period which it takes to double the value of the growing variable (called the doubling time) is a constant regardless of the current level of the variable. Thus, it will take just as long for the variable to double from 1 to 2 as it does to double from 1,000 to 2,000, or from 1,000,000 to 2,000,000. Hence, while the variables in Figure 4.2d are growing at a steady exponential rate during the entire 200 year period, because of the large vertical scale necessary for the graph in order to show the values at the end of the period, it is not possible to see the growth during the early part of the period.

4.2 Goal Seeking

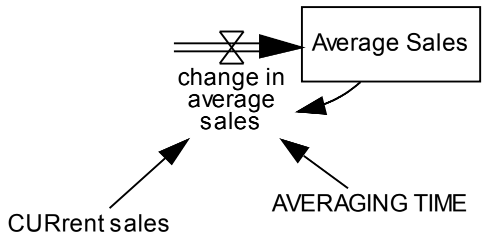
Figure 4.1b displays goal seeking behavior in which a process variable is driven to a particular value. Figure 4.3 presents a process which displays this behavior. As CURrent sales change, the level of Average Sales moves to become the same as CURrent sales. However, it moves smoothly from its old value to the CURrent sales value, and this is the origin of the name Average Sales for this variable. (In fact, this structure can be used to implement the SMOOTH function which we have previously seen.)

Figure 4.3c and Figure 4.3d show what happens when CURrent sales takes a step up (in Figure 4.3c) or a step down (in Figure 4.3d). While it is somewhat hard to see in these graphs, CURrent sales is plotted with a solid line which jumps at time 10. Until that time, Average Sales have been the same as CURrent sales, then they diverge since it takes a while for Average Sales to smoothly move to again become the same as CURrent sales.

Equation 3 in Figure 4.3b shows the process which drives Average Sales toward the value of CURrent sales. If Average Sales are below CURrent sales, then there is flow into the Average Sales stock, while if Average Sales are above CURrent sales, then there is flow out of the Average Sales stock. In either case, the flow continues as long as Average Sales differs from CURrent sales.

The rate at which the flow occurs depends on the constant AVERAGING TIME. The larger the value of this constant, the slower the flow into or out of Average Sales, and hence the longer it takes to bring the value of Average Sales to that of CURrent sales.

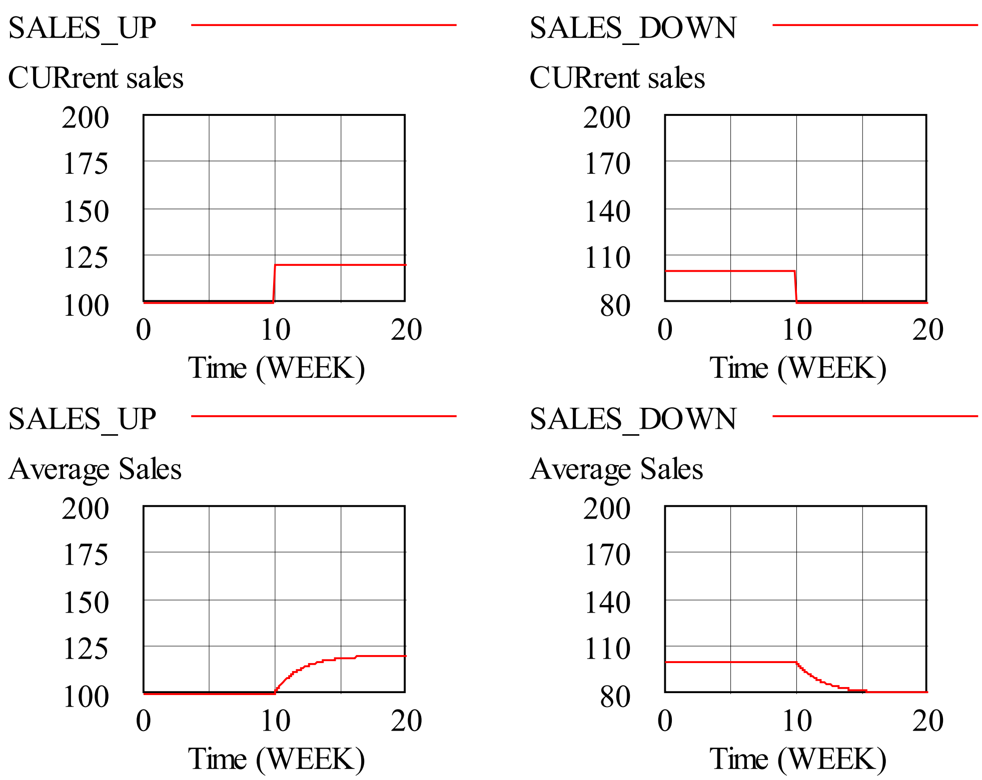
It is possible to solve the equations for a goal seeking process to show that the equation for the curve of the variable moving toward a goal (Average Sales in Figure 4.3) has an exponential shape. However, as with exponential growth, a goal seeking process is often only a part of a larger process for which it is not



a. Stock and flow diagram

```
(1)  Average Sales = INTEG(change in average sales,100)
(2)  AVERAGING TIME = 2
(3)  change in average sales = (CURrent sales-Average Sales)
     /AVERAGING TIME
(4)  CURrent sales = 100+STEP(20,10)
(5)  FINAL TIME  = 20
(6)  INITIAL TIME  = 0
(7)  SAVEPER  = TIME STEP
(8)  TIME STEP  = 0.0625
```

b. Vensim equations



c. Sales move up

d. Sales move down

Figure 4.3 *Goal seeking process*

possible to obtain a simple solution, and thus we show the simulation equations for this process.

Note that the process shown in Figure 4.4 is a negative feedback process. As the value of change in average sales increases, this causes an increase in the value of Average Sales, which in turn leads to a decrease in the value of change in average sales.

4.3 S-shaped Growth

Exponential growth can be exhilarating if it is occurring for something that you makes you money. The future prospects can seem endlessly bright, with things just getting better and better at an ever increasing rate. However, there are usually limits to this growth lurking somewhere in the background, and when these take effect the exponential growth turns into goal seeking behavior, as shown in Figure 4.1c.

Figure 4.4 shows a business process structure which can lead to this s-shaped growth pattern. This illustrates a possible structure for the sale of some sort of durable good for which word of mouth from current users is the source of new sales. This might be called a contagion model of sales being a user of the product is contagious to other people! We assume that there is a specified INITIAL TOTAL RELEVANT POPULATION of potential customers for the product. (This is the limit that will ultimately stop growth in Actual Customers.) At any point in time, there is a total of Potential Customers of potential users who have not yet bought the product.

Visualize the process of someone in the Potential Customers group being converted into an Actual Customer as follows: The two groups of people who are in the Actual Customers group and in the Potential Customers group circulate among the larger general population and from time to time they make contact. When they make contact, there is some chance that the comments of the person who is an Actual Customer will cause the person who is in the Potential Customers to buy the product.

The model shown in Figure 4.4 assumes that for each such contact between a person in the Actual Customers and a person in the susceptible population there will be a number of sales equal to SALES PER CONTACT, which will probably be less than one in most realistic settings. The number of sales per unit of time will be equal to SALES PER CONTACT times the number of contacts per unit of time between persons in the Actual Customers and Potential Customers groups. But with the assumed random contacts between persons in the two groups, the number of contacts per unit time will be proportional to both the size of the Actual Customers group and the size of the Potential Customers group. Hence sales is proportional to the product of Actual Customers and Potential Customers. The proportionality constant is called BASE CONTACT RATE in Figure 4.4, and it represents the number of contacts per unit time when each of the two groups has a size equal to one. (That is, it is the number of contacts per unit time between any specified member of the Actual Customers group and any specified member of the Potential Customers group.)

The argument in the last paragraph for a multiplicative form for the sales equation (as shown in equation 6 of Figure 4.4b) was somewhat informal. A more formal argument can be made by using probability theory. Select a short enough period of time so that at most one contact can occur between any persons in the Actual Customers group and the Potential Customers group regardless of how large these groups are. Then assume that the probability that any specified member of the Actual Customers group will contact any specified member of the Potential Customers group during this period is some (unspecified) number p. Then, if this probability is small enough (which we can make it by reducing the length of the time period considered), the probability that the specified member of the Actual Customers group will contact *any* member of the susceptible population is equal to $p \times$ Potential Customers.

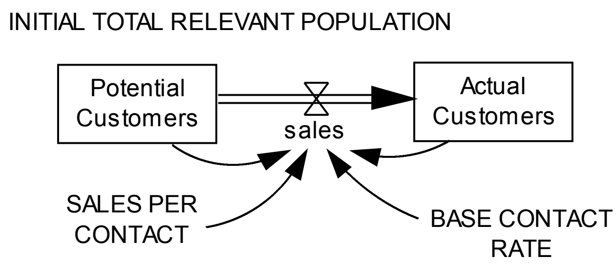
Assuming that this probability is small enough for any individual member of the Actual Customers population, then the probability that *any* member of the Actual Customers population will contact a member of the Potential Customers population is just this probability times the number of members in the Actual Customers population, or

$$p \times \text{susceptible population} \times \text{Actual Customers}$$

Assuming the interaction process between the two groups is a Poisson process and the probability of a successful interaction (that is, a sale) is fixed, then the sale process is a random erasure process on a Poisson process and hence is also a Poisson process. Thus, the expected number of sales per unit time is proportional to the probability expression above, and hence to the product of Actual Customers and susceptible population. This is the form assumed in equation 6 of Figure 4.4b.

Figure 4.4c shows the resulting pattern for the number of Actual Customer, as well as the sales. This s-shaped pattern is seen with many new products. First the process grows exponentially, and then it levels off. Sales also grow exponentially for a while, and then they decline. This can be a difficult process to manage because the limit to growth is often not obvious while the exponential growth is under way. For example, when a new consumer product like the compact disk player is introduced, what is the INITIAL TOTAL RELEVANT POPULATION of possible customers for the product? The difference between a smash hit like the compact disk player and a dud like quadraphonic high fidelity sound systems can be hard to predict.

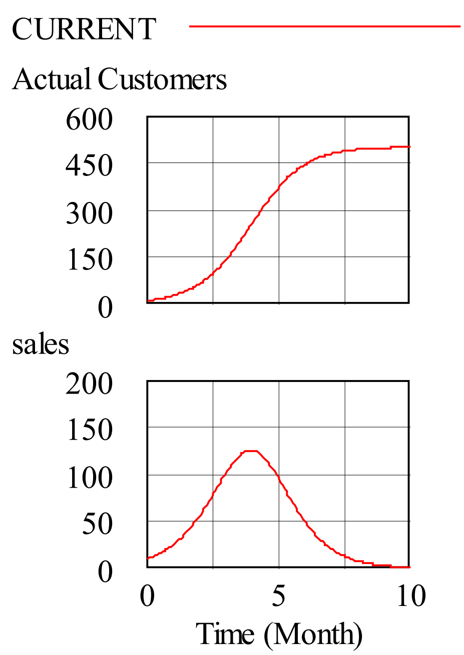
Note that there are two feedback loops, one positive and one negative, that involve the variable sales in the Figure 4.5a diagram. The positive loop involves sales and Actual Customers. The negative loop involves sales and Potential Customers. At first the positive loop dominates, but later the negative loop comes to dominate. (There is another feedback loop through the initial condition on Potential Customers, which depends on Actual Customers. However, this is not active once the process starts running.)



a. Stock and flow diagram

```
(01)  Actual Customers = INTEG(sales, 10)
(02)  BASE CONTACT RATE = 0.02
(03)  FINAL TIME  = 10
(04)  INITIAL TIME  = 0
(05)  Potential Customers = INTEG(-sales,
        INITIAL TOTAL RELEVANT POPULATION - Actual Customers)
(06)  sales = BASE CONTACT RATE * SALES PER CONTACT
              * Actual Customers * Potential Customers
(07)  SALES PER CONTACT = 0.1
(08)  SAVEPER  = TIME STEP
(09)  TIME STEP  = 0.0625
(10)  INITIAL TOTAL RELEVANT POPULATION = 500
```

b. Vensim equations



c. Customer and sales performance

Figure 4.4 *S-shaped growth process*

4.4 S-shaped Growth Followed by Decline

Figure 4.5 shows a process model for a variation on s-shaped growth where the leveling off process is followed by decline. In this process, it is assumed that some Actual Customers and some Potential Customers permanently quit. Such a process might make sense for a new fad durable good which comes on the market. In such a situation, there may be a large INITIAL TOTAL RELEVANT POPULATION of possible customers but some of those who purchase the product and become Actual Customers may lose interest in the product and cease to discuss it with Potential Customers. Similarly, some Potential Customers lose interest before they are contacted by Actual Customers. Gradually both sales and use of the product will decline.

In equation 5 of Figure 4.5, the quitting processes for both Potential Customers and Actual Customers are shown as exponential growth processes running in reverse. That is, the number of Actual Customers *leaving* is proportional to the number of Actual Customers rather than the number *arriving*, as in a standard exponential growth process. Similarly, the number of Potential Customers leaving is proportional to the number of Potential Customers. This type of departure process also can be viewed as a balancing process with a goal of zero, and it is sometimes called *exponential decline* or *exponential decay*. From Figure 4.5c, we see that this exponential decline process eventually leads to a decline in the number of Actual Customers.

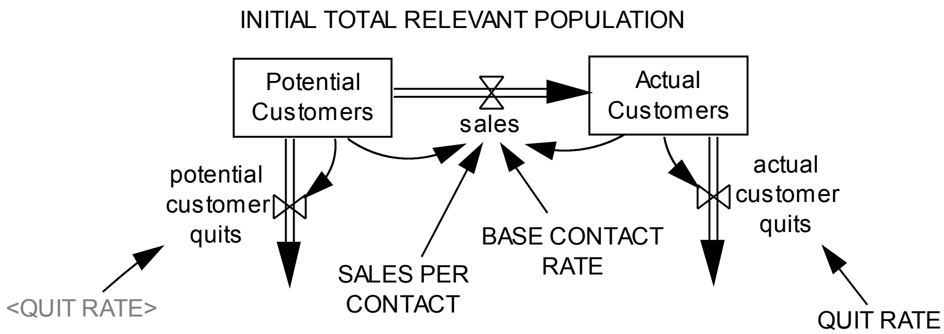
4.5 Oscillating Process

The Figure 4.6a stock and flow diagram is a simplified version of a production-distribution process. In this process, the retailer orders to the factory depend on both the retail sales and the Retail Inventory level. The factory production process is shown as immediately producing to fulfill the retailer orders, but there is a delay in the retailer receiving the product because of shipping delays.

In this process, RETail sales are 100 units per week until week 5, at which point they jump to 120 units and remain there for the rest of the simulation run. We see from Figure 4.6c that there are substantial oscillations in key variables of the process.

Unless there are very unusual flow equations, there must be at least two stocks in a process for the process to oscillate. Furthermore, the degree of oscillation is usually impacted by the delays in the process. The important role of stocks and delays in causing oscillation is one of the factors behind moves to just in time production systems and computer-based ordering processes. These approaches can reduce the stocks in a process and also can reduce delays.

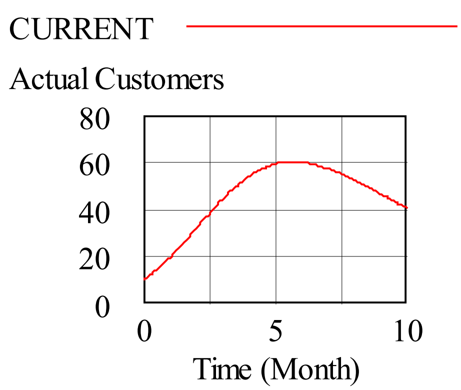
Figure 4.7 illustrates another aspect of oscillating systems. The process in Figure 4.7 is identical to that in Figure 4.6 except that the RETail sales function has been changed from a step to a sinusoid. Thus, sales are stable at 100 units per week until week 5, and then sales vary sinusoidally with an amplitude above and below 100 units per week of 20. The results for three different cycle lengths are



a. Stock and flow diagram

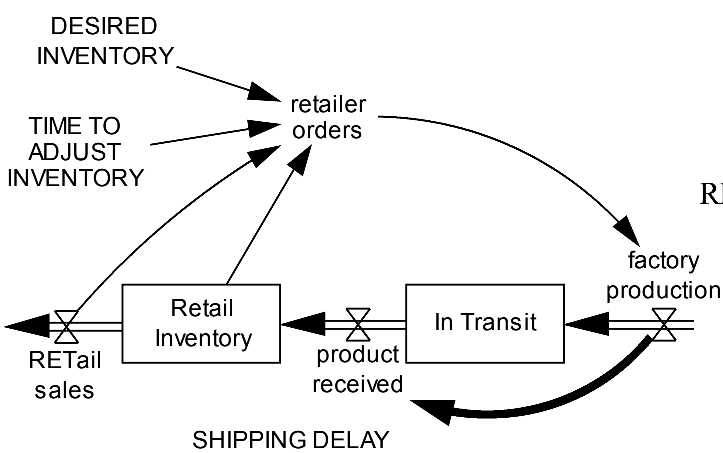
```
(01)  actual customer quits = QUIT RATE * Actual Customers
(02)  Actual Customers = INTEG(sales - actual customer quits, 10)
(03)  BASE CONTACT RATE = 0.02
(04)  FINAL TIME  = 10
(05)  INITIAL TIME  = 0
(06)  INITIAL TOTAL RELEVANT POPULATION = 500
(07)  potential customer quits = QUIT RATE * Potential Customers
(08)  Potential Customers = INTEG(-sales - potential customer quits,
        INITIAL TOTAL RELEVANT POPULATION - Actual Customers)
(09)  QUIT RATE = 0.2
(10)  sales = BASE CONTACT RATE * SALES PER CONTACT
              * Actual Customers * Potential Customers
(11)  SALES PER CONTACT = 0.1
(12)  SAVEPER  = TIME STEP
(13)  TIME STEP  = 0.0625
```

b. Vensim equations

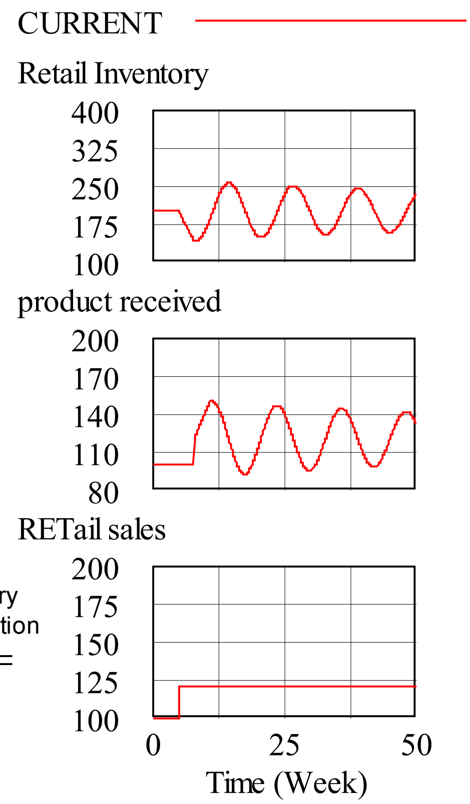


c. Customer performance

Figure 4.5 *S-shaped growth followed by decline*



a. Stock and flow diagram



c. Process oscillations

```
(01)  DESIRED INVENTORY = 200
(02)  factory production = retailer orders
(03)  FINAL TIME  = 50
(04)  In Transit = INTEG(factory production-orders received, 300)
(05)  INITIAL TIME  = 0
(06)  product received = DELAY FIXED(factory production,
                           SHIPPING DELAY, factory production)
(07)  Retail Inventory = INTEG(product received-retail sales, 200)
(08)  RETail sales = 100 + STEP(20, 5)
(09)  retailer orders = retail sales+ (DESIRED INVENTORY
                    - Retail Inventory) / TIME TO ADJUST INVENTORY
(10)  SAVEPER  = TIME STEP
(11)  SHIPPING DELAY = 3
(12)  TIME STEP  = 0.0625
(13)  TIME TO ADJUST INVENTORY = 2
```

b. Vensim equations

Figure 4.6 *Oscillating feedback process* shown in Figure 4.7c. The RUN4 results are for a cycle length of 4 weeks (that is, a monthly cycle). The RUN13 results are for a 13 week (that is, quarterly) cycle, and the RUN52 results are for a 52 week (that is, annual) cycle.

Notice that the amplitude of the variations in Retail Inventory and product received are different for the three different cycle lengths. The amplitude is considerably greater for the 13 week cycle than for either the 4 week or 52 week cycles. This is true even though the amplitude of the RETail sales is the same for each cycle length.

Now go back and examine the curves in Figure 4.6c which shows the response of this process to a step change in retail sales. Note in particular that the cycle length for the oscillations is around 12 weeks. A cycle length at which a process oscillates in response to a step input is called a *resonance* of the process, and the inverse of the cycle length is called a *resonant frequency.* Thus, a resonant frequency for this process is 1 12 = 0 0833 cycles per week.

A process will generally respond with greater amplitude to inputs which vary with a frequency that is at or near a resonant frequency. Thus, it is to be expected that the response shown in Figure 4.7c for the sinusoidal with a 13 week cycle is greater than the responses for the sinusoids with 4 and 52 week cycles.

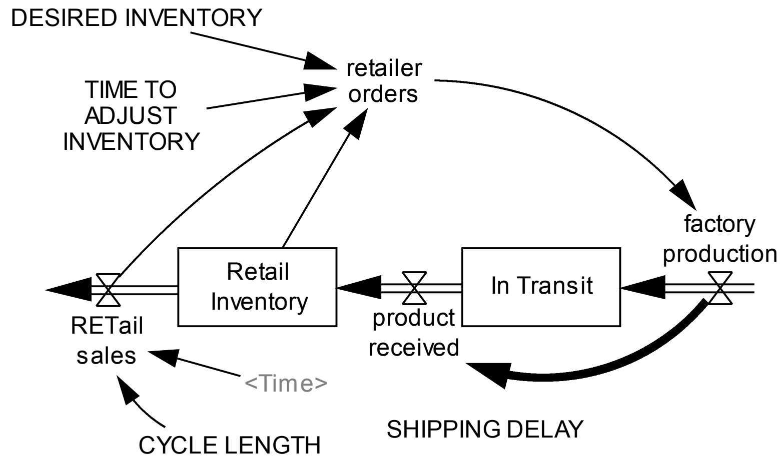
In engineered systems, an attempt is often made to keep the resonant frequencies considerably different from the usual variations that are found in operation. This is because of the large responses that such systems typically make to inputs near their resonant frequencies. This can be annoying, or even dangerous. (Have you ever noticed the short period of vibration that some planes go through just after takeoff? This is a resonance phenomena.)

Unfortunately, the resonant frequencies for many business processes are in the range of variations that are often found in practice. This has two undesirable aspects. First, it means that the amplitude of variations is greater than it might otherwise be. Second, it may lead managers to assume there are external causes for the variations. Suppose that in a particular process these oscillations have periods that are similar to some natural time period like a month, quarter, or year. In such a situation, it can be easy to assume that there is some external pattern that has such a period, and start to organize your process to such a cycle. This can make the oscillations worse. For example, consider the traditional yearly cycle in auto sales. Is that due to real variations in consumer demand, or is it created by the way that the auto companies manage their processes?

4.6 References

G. P. Richardson and A. L. Pugh III, *Introduction to System Dynamics Modeling with DYNAMO*, Productivity Press, Cambridge, Massachusetts, 1981.

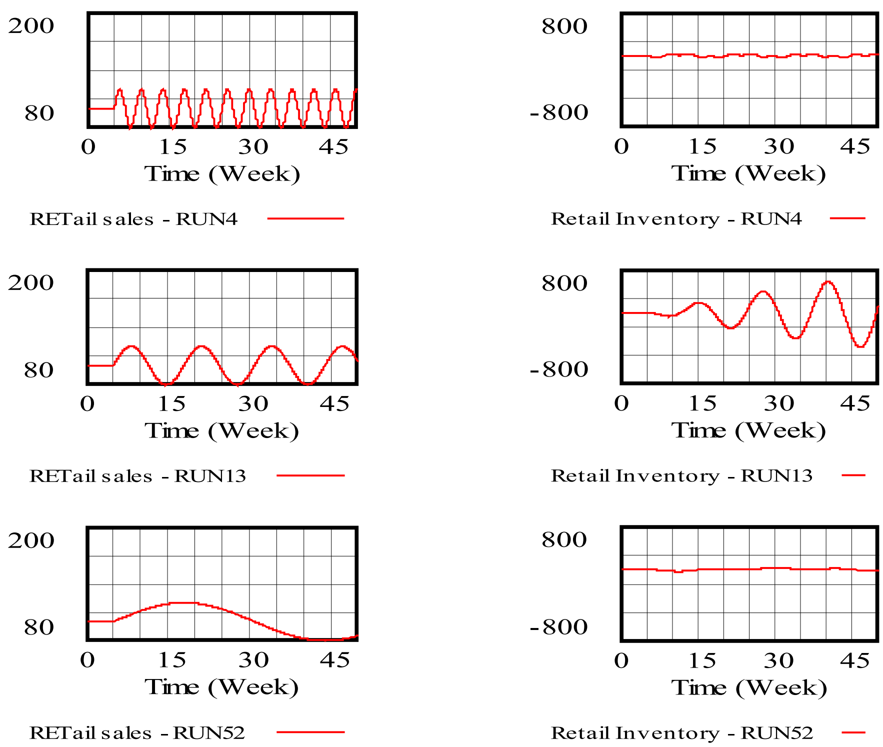
P. M. Senge, *The Fifth Discipline: The Art and Practice of the Learning Organization*, Doubleday Currency, New York, 1990.



a. Diagram

```
CYCLE LENGTH = 13
RETail sales
  = 100 + STEP(20, 5) * SIN(2 * 3.14159 * (Time-5) / CYCLE LENGTH)
```

b. Changes to Figure 4.6 Vensim equations



c. Process oscillations (4, 13, and 52 week cycles)

Figure 4.7 *Performance with oscillating retail sales*